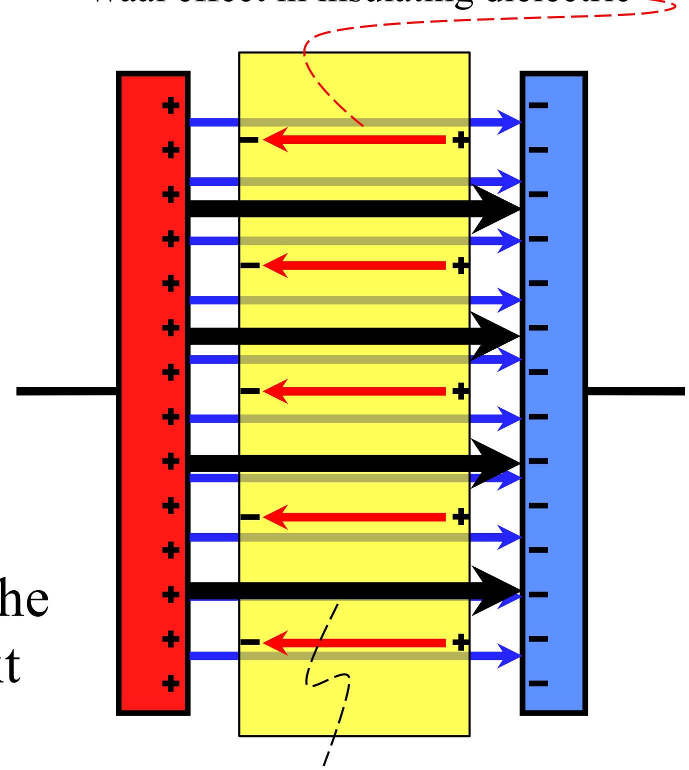
Dielectrics

Consider the charged, *parallel-plate capacitor* shown to the right (complete with its *E-fld*).



Placing an insulating material (called a *dielectric*) between the plates does a number of things.

1.) *The dielectric* experiences a van der Waal effect due to its presence in the electric field between the plates. This creates a reverse electric field that diminishes the net electric field across the plates (see sketch on next page).

2.) *With the net electric field* diminishing, the net electrical potential across the plates goes DOWN. As C=q/V, a diminishing of V means the capacitance goes UP.

3.) *Conceptually,* placing a dielectric between the plates effectively allows the plates to hold more charge per unit volt. This is why the capacitance increases when a dielectric is placed internal to the cap.

Net effect: For the charged, *parallel-plate capacitor* shown to the right.

1.) *The capacitance* of a capacitor *with a dielectric* between its plates will equal:

$$C_{\text{with dielectric}} = \kappa C_{\text{without dielectric}},$$

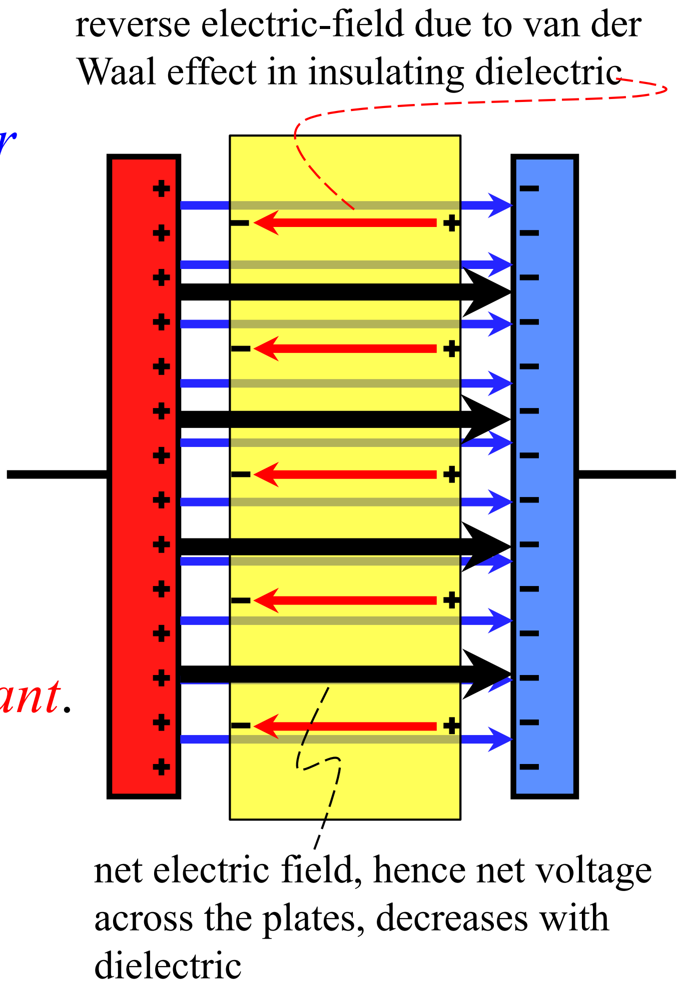
where κ, sometimes characterized as ε_d, is the proportionality constant called the *dielectric constant*.

Note 1: This means there are three ways to increase a capacitor's value:

1.) increase the plate area.

2.) bring the plates closer together.

3.) place an insulating *dielectric* between the plates.



Dielectrics Summary

- So far, all the capacitors we've considered have had air in between the plates. What if another material is in there?
- A **dielectric** is a material (e.g. rubber, plastic, waxed paper) that **increases the capacitance** when placed between the plates.
  - Imagine a charged capacitor of capacitance C_0. Because it's charged, it has a voltage between the plates of V_0 and charge on a plate Q_0.
  - If the dielectric is inserted into the capacitor, the charge doesn't change (nothing allowed the plates to equalize), but the voltage is observed to decrease by a factor **κ**, so $V_{new} = V_0/\boldsymbol{\kappa}$.
  - Therefore, we can say: $C_{new} = \frac{Q_0}{V_0/\kappa} = \kappa C_0$ where **κ** is called the **dielectric constant**, and is unique to a substance (table on p. 575)
    - **κ** for air is 1.00059 (or, basically, 1)
  - So, for any parallel plate capacitor, we can say: $\boxed{C = \kappa \epsilon_0 \frac{A}{d}}$

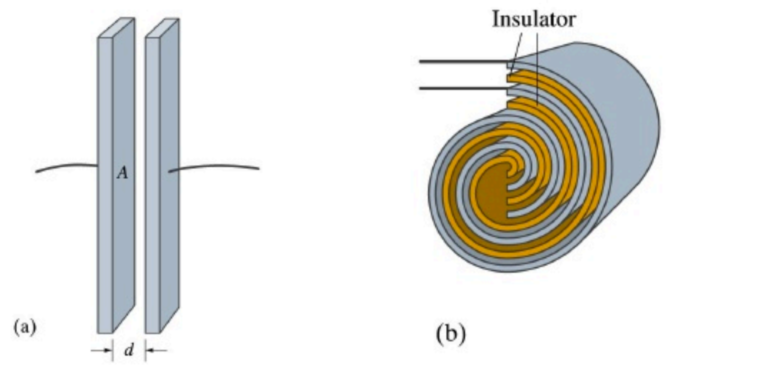
Dielectrics

- What's the point of a dielectric?
  - Increasing the capacitance (duh). But how…?
- With a dielectric inserted…
  - The plates can be pushed closer together, increasing capacitance. Why?

    --A good dielectric has a greater "breakdown voltage" than air. Remember the Christmas lights? Air will ionize and allow electricity to conduct if the electric field is strong enough. With a dielectric in the way, the stronger electric field due to the plates being closer together can't break down the dielectric as easily.

    --Also recall that capacitors don't stay charged forever! There will be a tiiiiiiiny trickle of charge between the plates, so over time the plates will equalize. A dielectric extends this time even more.

- The capacitor can be rolled up into a tiny size, with the dielectric keeping the plates from touching.



Parallel-Plate Capacitor   Cylindrical Capacitor

A cylindrical capacitor is a parallel-plate capacitor that has been rolled up with an insulating layer between the plates.

Practice question (16.49)

- The voltage across an air filled capacitor is 85 volts. With a dielectric between the plates, the voltage is 25 volts.

  a.) What is the dielectric constant? Can you tell what the dielectric is?

$$\kappa = \frac{V_0}{V_{new}} = 3.4 \rightarrow \text{plexiglas}$$

- b.) If the dielectric doesn't completely fill the space, what can you conclude about the voltage across the plates?

It's somewhere between 25 V and 85 V

Dielectric Constants at 20°C

Material	Dielectric Constant
Vacuum	1
Glass	5-10
Mica	3-6
Mylar	3.1
Neoprene	6.70
Plexiglas	3.40
Polyethylene	2.25
Polyvinyl chloride	3.18
Teflon	2.1
Germanium	16
Strontiun titanate	310
Titanium dioxide (rutile)	173 perp 86 para
Water	80.4
Glycerin	42.5
Liquid ammonia(-78°C	25
Benzene	2.284
Air(1 atm)	1.00059
Air(100 atm)	1.0548

The dielectric constant k is the relative permittivity of a dielectric material. It is an important parameter in characterizing capacitors. It is unfortunate that the same symbol k is often used for Coulomb's constant, so one must be careful of this possible confusion. It is more typical of physics texts to use the form $1/4\pi\varepsilon_0$ for Coulomb's constant.

Index

Tables

Reference Sears, Zemansky, Young Table 27-1

Electric and magnetic constants

Dielectric constants of semiconductors

HyperPhysics***** Electricity and Magnetism

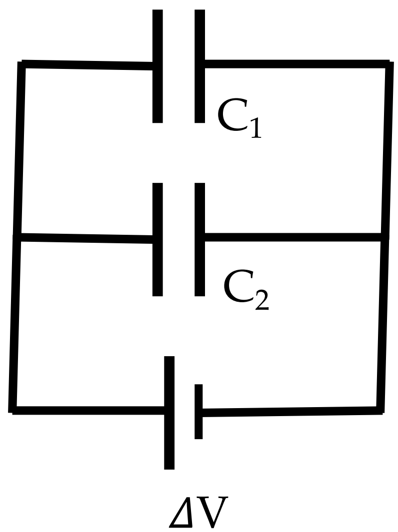
Go Back

Capacitors in combination

- Just like resistors, capacitors can be put in combinations (series and parallel) to change the equivalent capacitance of a circuit.
- The equations for series and parallel for capacitors is **reversed** from those for resistors! Here's why:

These parallel capacitors are both connected to the battery. This means the voltage across each capacitor is equal to that of the battery, or $V_{cap} = V_{batt.}$

$\Delta V_1 = \Delta V_2 = \Delta V_{batt}$



Each capacitor can store some amount of charge Q, and we can say $Q_{total} = Q_1 + Q_2$. Combining that with the voltage expression, we get:

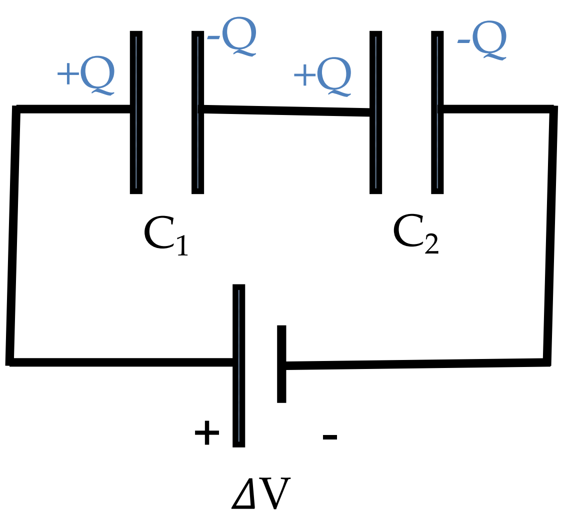
$$Q_{tot} = Q_1 + Q_2$$
$$C_{eq}\,\Delta V = C_1\,\Delta V + C_2\,\Delta V$$
$$\boxed{\mathbf{C_{eq,parallel} = C_1 + C_2}}$$

Key point about capacitors in parallel: the voltage across each capacitor is the same!

Capacitors in combination

- How about in series?

Recall that the plates of a capacitor have equal and opposite charges. As the capacitors begin charging, one plate gains charge and other loses. For this particular circuit, the left-hand plates of each capacitor acquire charge +Q, and the right-hand plates acquire charge -Q. This also means that the Q of the equivalent circuit is the same as the Q on either capacitor! Why are these things true?



Key point about capacitors in series:
all capacitors have the same net Q

Q on each capacitor is the same, but the voltage is not - by Kirchhoff's loop rule, we have: $\Delta V = \Delta V_1 + \Delta V_2$

Therefore, we can say:

$$\frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2}$$

Simplifying:

$$\boxed{\frac{1}{C_{eq,series}} = \frac{1}{C_1} + \frac{1}{C_2}}$$

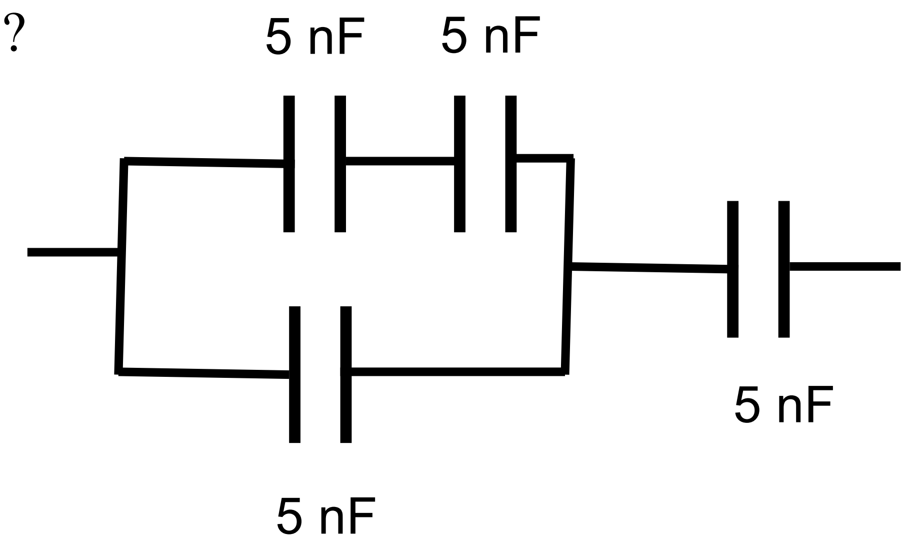
Mixed combinations

- Analyzing a complex circuit with capacitors in both series and parallel is just like analyzing a complex circuit of resistors!
  - Use the series and parallel rules to simplify and redraw the circuit until you have a single equivalent capacitance
  - Remember rules about whether voltage is equal across the capacitors (parallel) or charge is equal on all capacitors (series)
  - Use Kirchhoff's rules if necessary around a loop.

$$\mathrm{C}_{\mathrm{eq,parallel}} = \mathrm{C}_1 + \mathrm{C}_2 + \cdots \qquad \frac{1}{\mathrm{C}_{\mathrm{eq,series}}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots$$

Example

- What is the equivalent capacitance?



Solution

Start with the series branch:

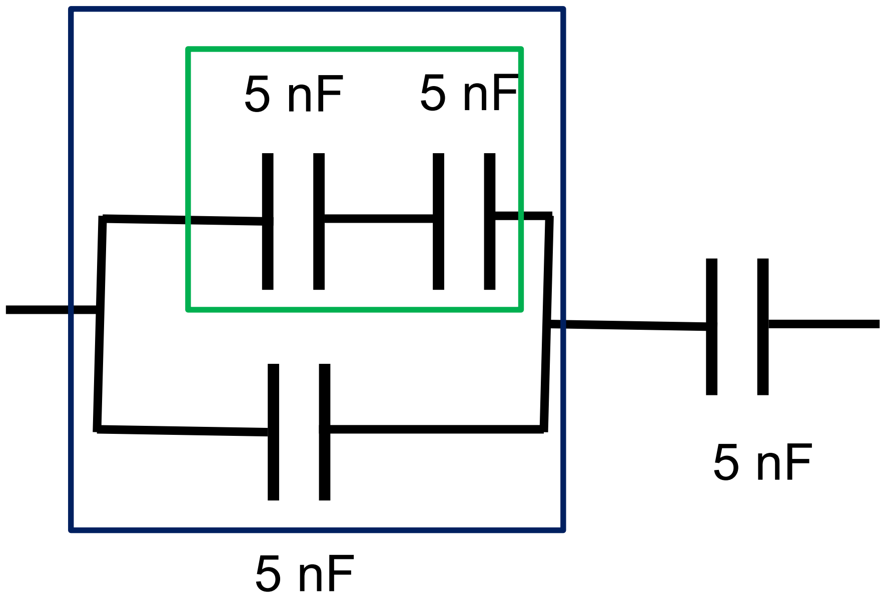
$$\frac{1}{C_{eq}} = \frac{1}{5} + \frac{1}{5} \rightarrow C_{eq} = \frac{5}{2}\text{nF}$$

Now that's in parallel with the other branch:

$$C_{eq} = \frac{5}{2} + 5 = 7.5 \text{ nF}$$

Finally, that is in series with the lone capacitor on the right:

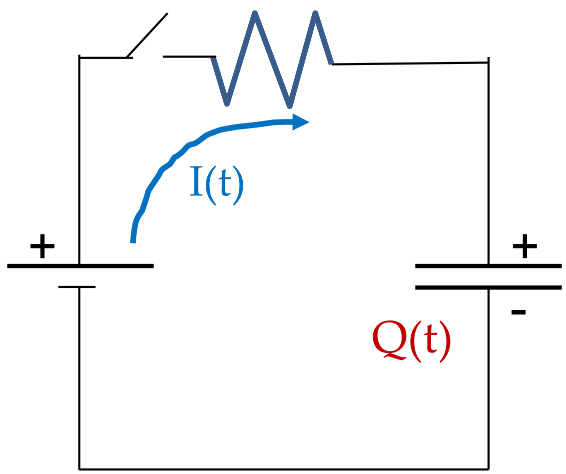
$$\frac{1}{C_{eq}} = \frac{1}{7.5} + \frac{1}{5} \rightarrow C_{eq} = 3 \text{ nF}$$



RC circuits

- This type of circuit, with a DC power source, capacitor, and resistor, is known as an **RC circuit**.
- The question is: when you connect this circuit, how fast will it act? How about during discharging? To answer that question, we look at something called the **RC time constant**.

❑ As we close the switch, current will flow and the capacitor will begin to charge. We can use Kirchhoff's loop rule around the circuit here at any given time *t*:



$$V_0 - I(t)R - \frac{Q(t)}{C} = 0$$

Then divide by R and rearrange:

$$I(t) + \frac{1}{RC}Q(t) = \frac{V_0}{R}$$

Then remembering that I(t) = d(Q(t))/dt:

$$\frac{d(Q(t))}{dt} + \left(\frac{1}{RC}\right)Q(t) = \frac{V_0}{R}$$

RC equations - charging capacitor

$$\frac{d(Q(t))}{dt} + \left(\frac{1}{RC}\right) Q(t) = \frac{V_0}{R}$$

The solution to this equation is exponential:

$$\boxed{Q(t) = Q_{max}(1 - e^{-t/RC})}$$

This tells us that at $t = 0$, $e^0 = 1$ so $Q = 0$ -- just what we'd expect. No charge has built up. At $t =$ infinity, $Q = Q_{max}$ -- also what we'd expect.

And:

$$I(t) = \frac{dq}{dt} = \left(-\frac{1}{RC}\right) Q_{max}(-e^{-t/RC}) = \left(\frac{1}{R}\right)\left(\frac{Q_{max}}{C}\right)(e^{-t/RC})$$

$$\boxed{I(t) = \frac{V_0}{R} e^{-t/RC} = I_0 e^{-t/RC}}$$

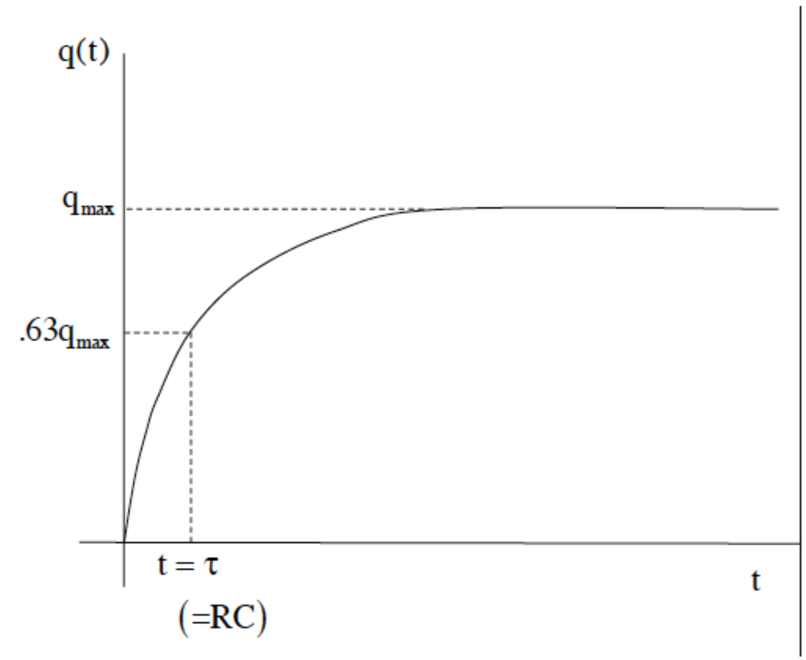
Same checks: at $t = 0$, $I = I_0$ or maximum current. At $t =$ infinity, $I =$ 0 as we'd expect, because the capacitor is charged.

Time constant

- We can see from these equations that it takes an infinite time to fully charge the capacitor. However, in practicality, the capacitor <u>does</u> charge in a finite time.
- The RC term in the exponent of both equations is called the **time constant** of the circuit, given the symbol $\tau = RC$.
- When $t = \tau$, we get $Q_{max}\left(1 - e^{-\frac{RC}{RC}}\right) = Q_{max}(1 - .3679) = .63Q_{max}$

This means after a period of time equal to one time constant, the charge on the capacitor is 63% of the maximum charge. This is true no matter what the time constant is!

If you have a graph of Q vs t, you can look at when Q is 63% of maximum, and find the time constant!



Discharging capacitor

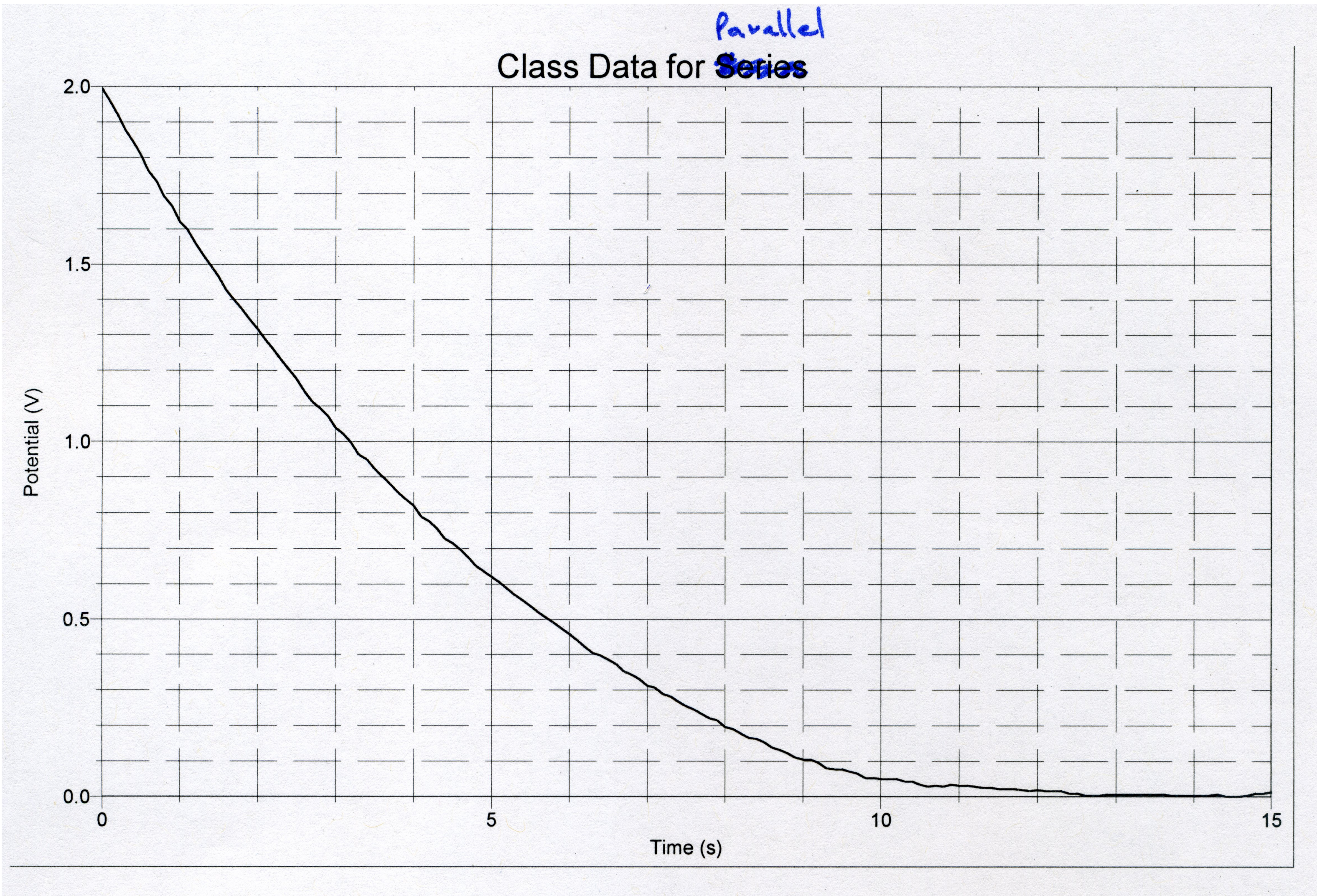
- For a discharging capacitor, the reverse is true. In the amount of time equal to one time constant, the capacitor will lose 63% of its initial charge. This makes the equation $Q(t) = Q_{max}(e^{-t/RC})$

- The current, in a discharging capacitor, however, has the same behavior: maximum current at $t = 0$, decreasing as time goes on. That equation stays the same.

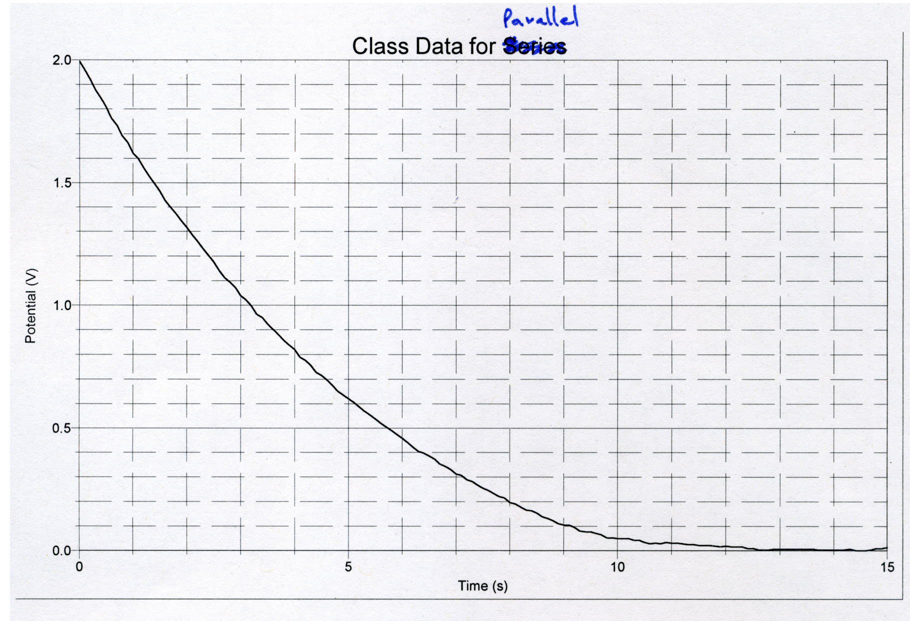
- What would the graph look like?

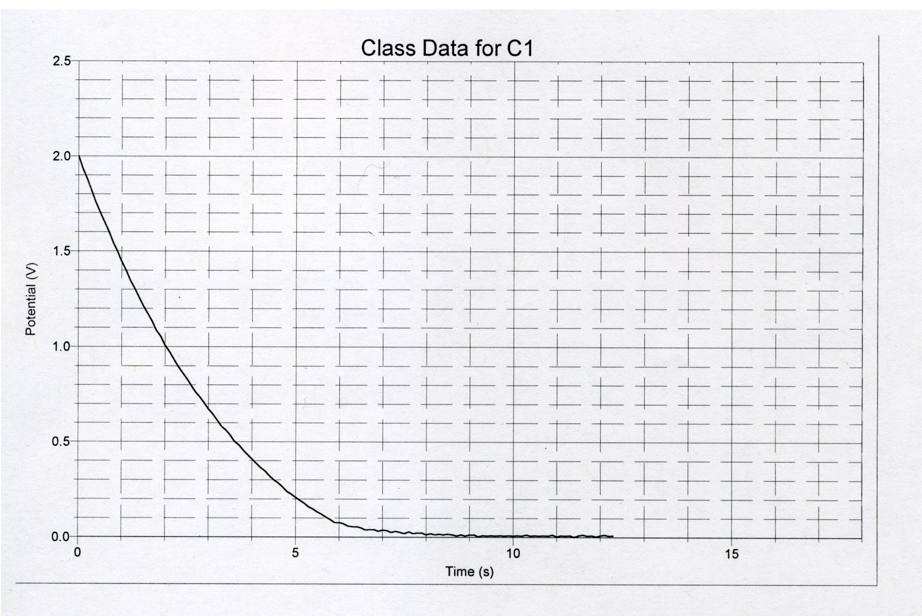
Discharging capacitors

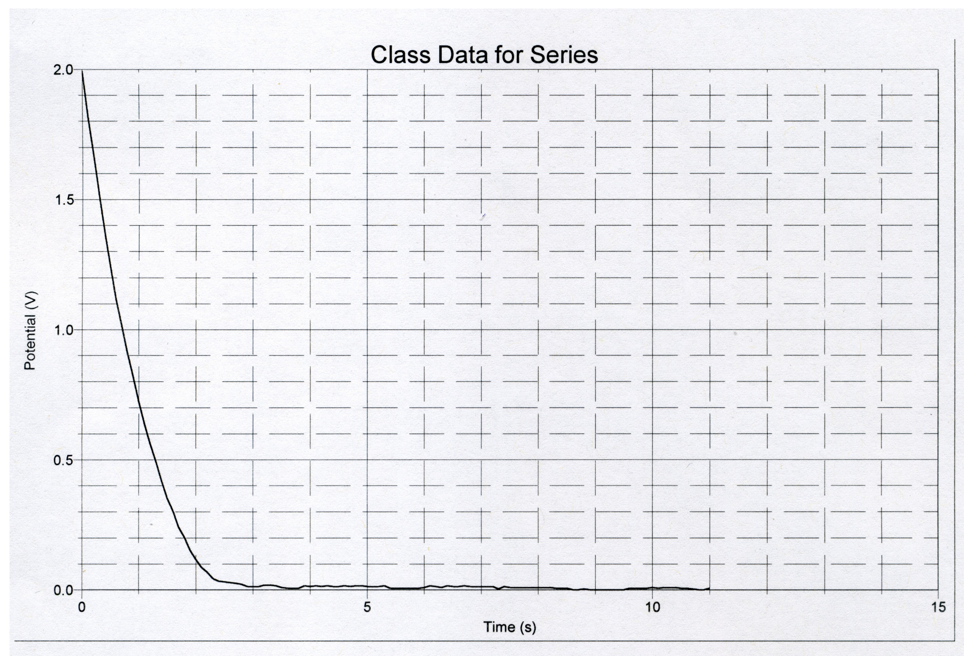
Let's try it - what's the time constant for this capacitor? (remember Q and V are proportional)



These are discharge graphs for three different capacitances (same resistor in each case). Which cap must have been largest?



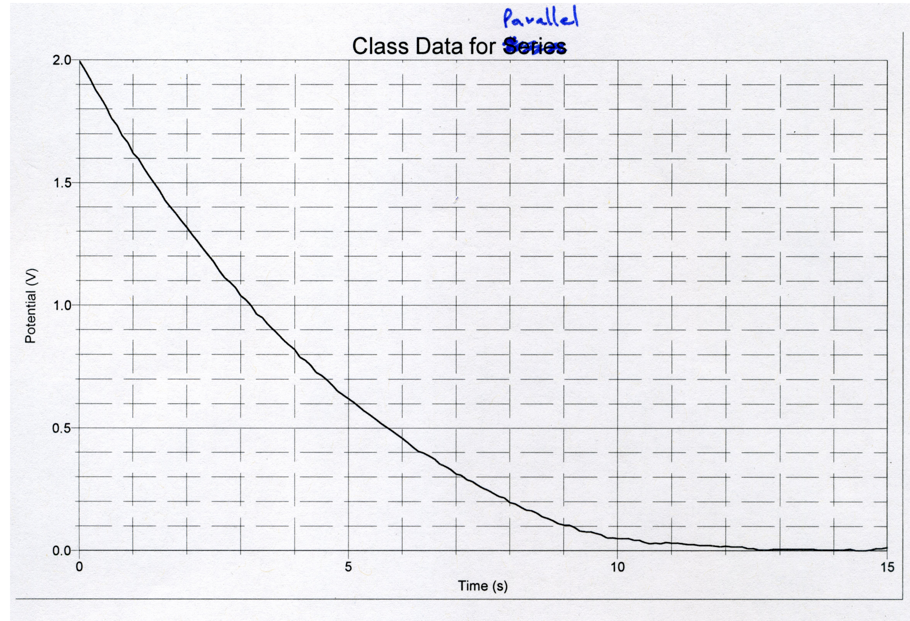


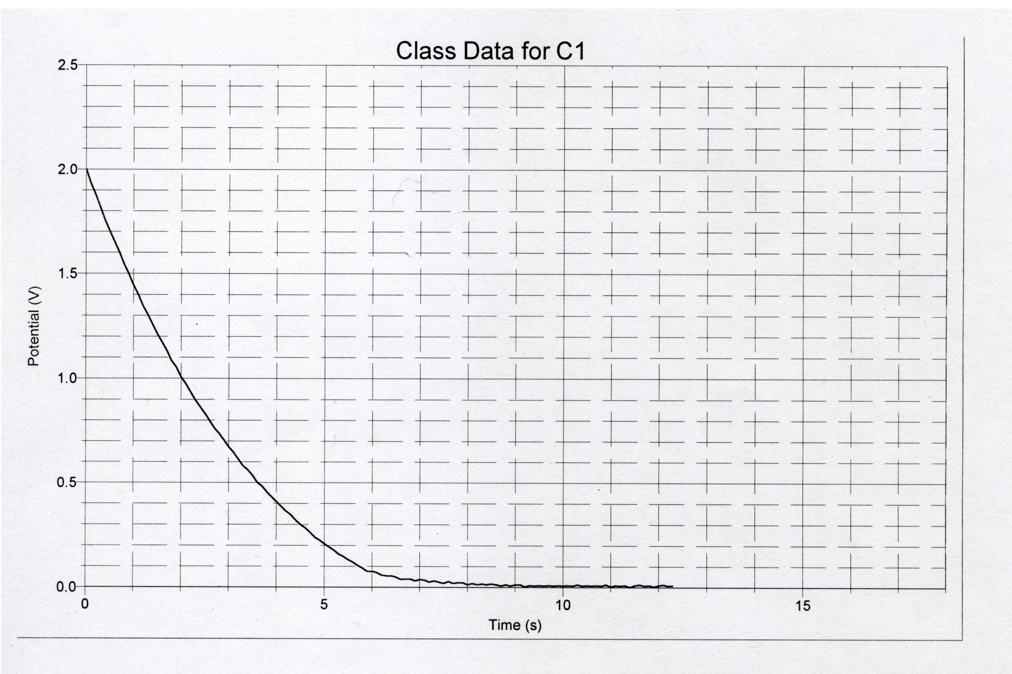


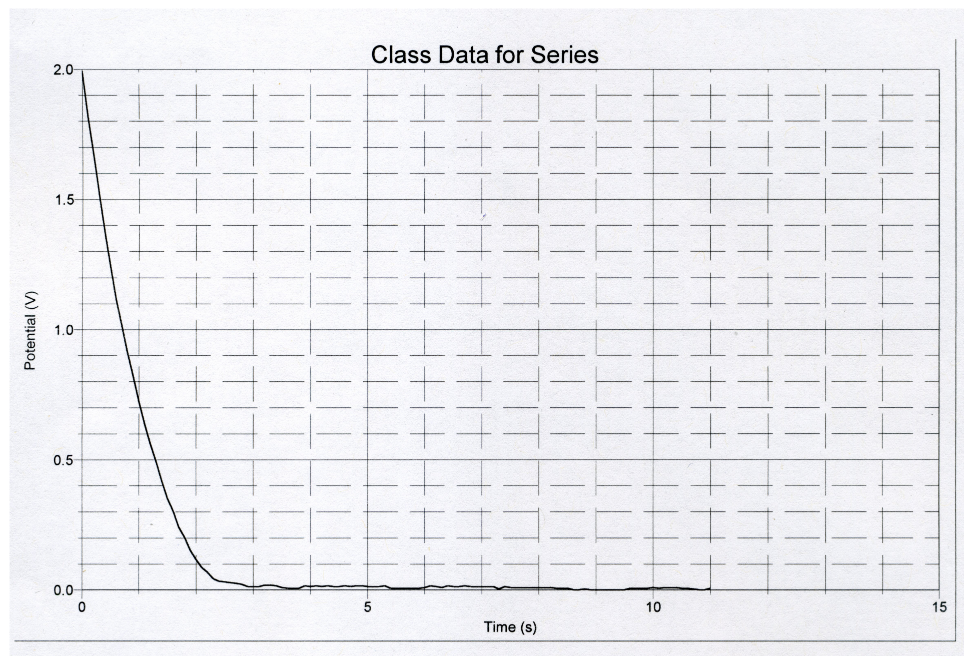
The resistor and capacitor values for one of the graphs is given. With which graph do they go?

$C_{series} = 29.2\ \mu F$

$R = 40.5\ k\Omega$







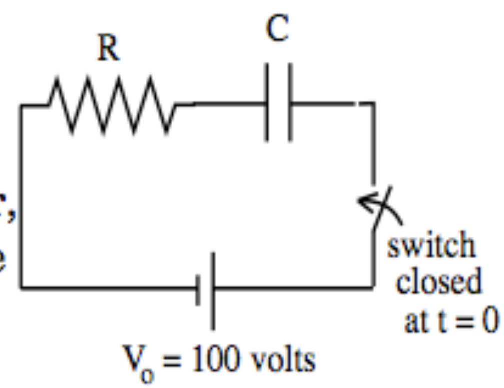
RC circuit refresher

- We looked at a circuit containing a resistor and capacitor. What happens when they're connected? How can we model the behavior of the circuit in terms of Q(t) and I(t) for both charging and discharging?

- We ended up with two relationships for Q(t) depending on charging vs. discharging (which is which, and why?):
- $Q(t) = Q_{max}(1-e^{-t/RC})$ or $Q(t) = Q_{max}e^{-t/RC}$
- Current is always $I(t) = I_0e^{-t/RC}$ - why?
- What is special when t = RC? What is it called? What does it tell us?

Two quick checks

14.2) What do capacitors (often referred to as *caps*) generally do in DC circuits? Give an example.



14.3) A 10^{-6} *farad* capacitor is in series with a 10^4 *ohm* resistor, a battery whose voltage is $V_o = 100$ *volts*, and a switch. Assume the capacitor is initially uncharged and the switch is thrown at $t = 0$.

a.) The capacitance value tells you something that is always true no matter what the voltage across the capacitor happens to be. What does it tell you?

b.) What is the initial current in the circuit?

c.) What is the circuit's current after a long period of time?

d.) How much charge will the capacitor hold when fully charged?

e.) How much energy is wrapped up in the capacitor when fully charged?

continued

f.) Where is the energy stored in the capacitor?

g.) You are told that the time constant for the system is 10^{-2} seconds.

i.) What does that tell you about the system?

ii.) How much charge will be associated with the capacitor after at time equal to one time constant?

iii.) Where will the charge alluded to in *Part g-ii* be found?

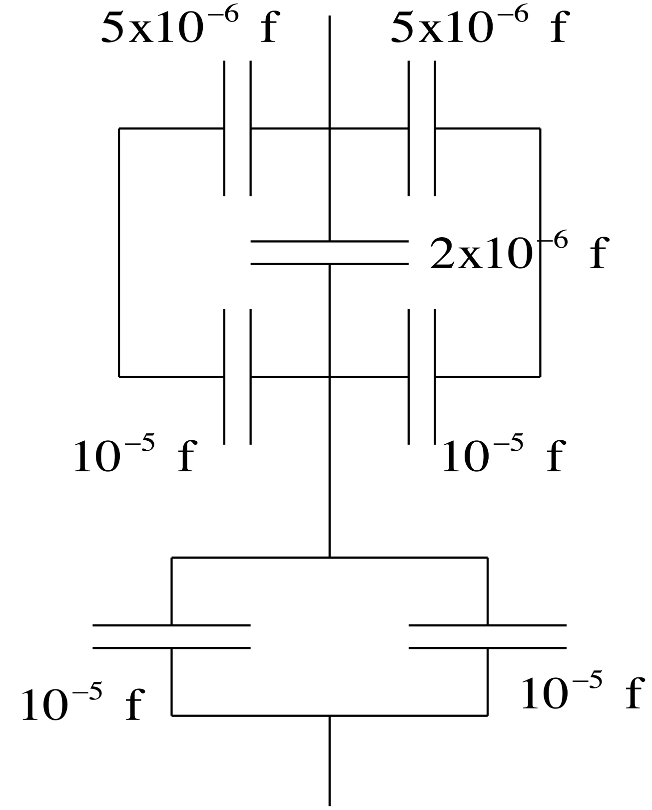
h.) After a very long time, the switch is opened. What happens to the capacitor? Will it hold its charge forever?

i.) At $t = 1$ *second*, the current is i_1. At $t = 2$ *seconds*, the current is i_2. At $t = 4$ *seconds*, the current is i_4, and at $t = 8$ *seconds*, the current is i_8. Is i_2/i_1 going to give you the same ratio as i_8/i_4?

Solutions are in Mr Fletcher's book online

Problem 16.42

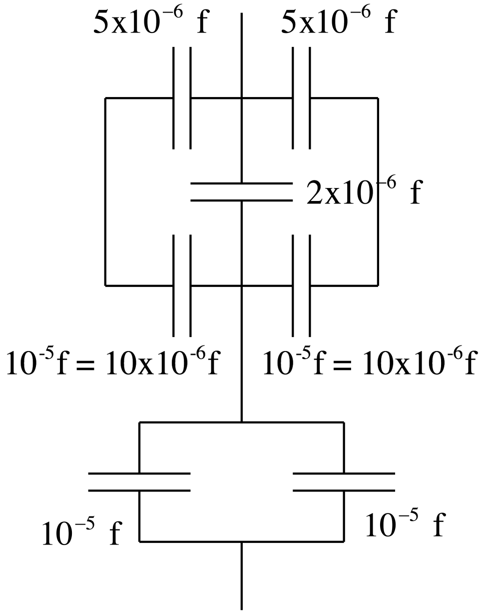
What is the equivalent capacitance for the circuit to the right?



Solution is on MyPoly

Problem 16.42

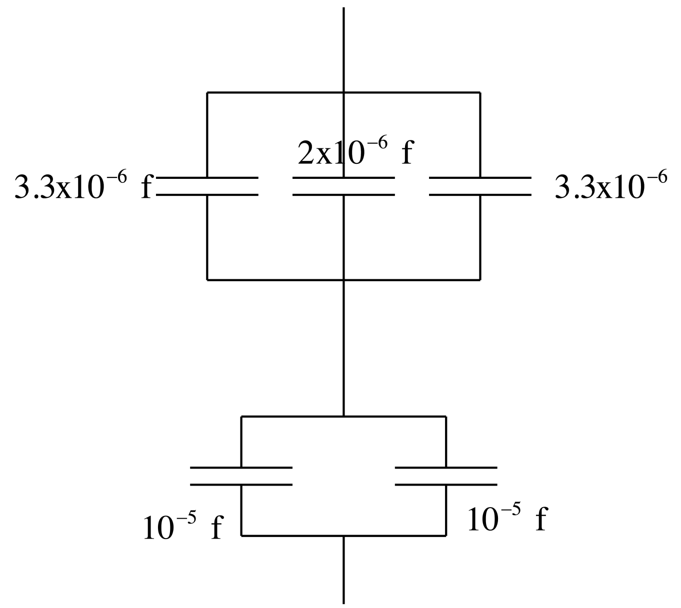
What is the equivalent capacitance of the circuit?



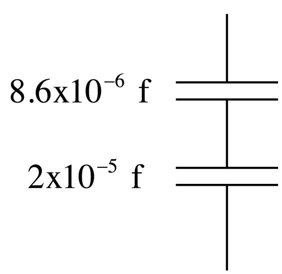
(note that the two caps in each upper side-shoot are in series)

$\Rightarrow$

(note that the two caps in the lower section are in parallel)



$\Swarrow$



$\Rightarrow$

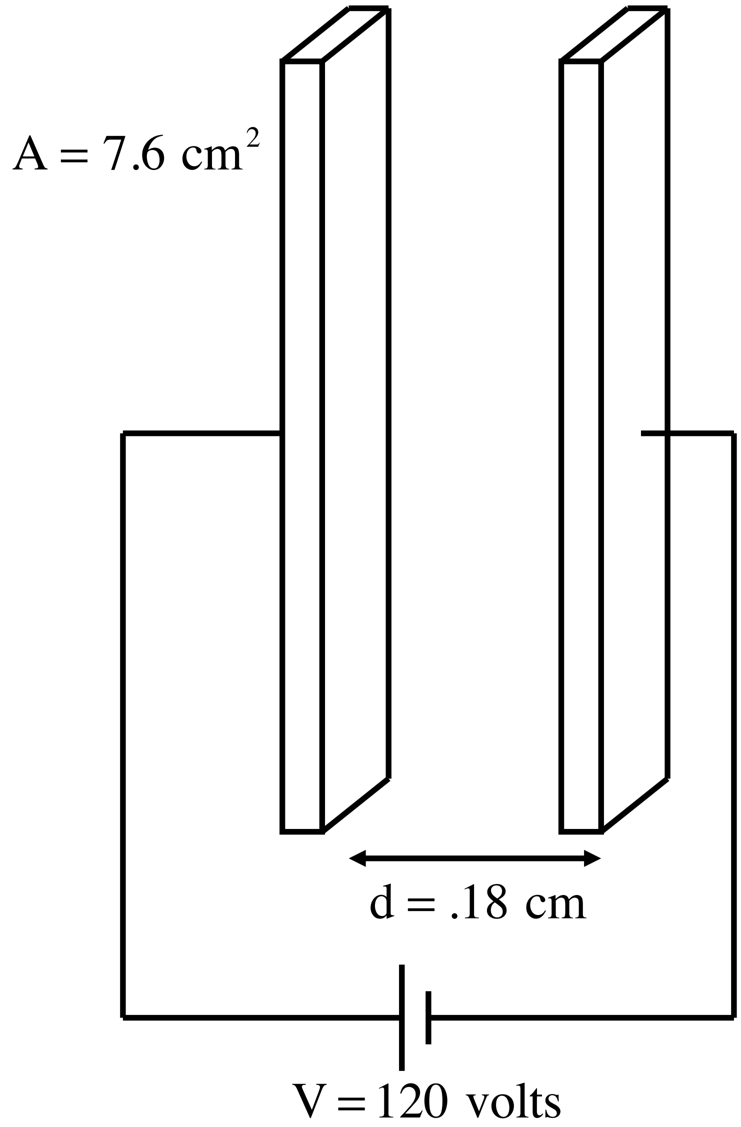
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\frac{1}{C_{eq}} = \frac{1}{8.6x10^{-6}\ f} + \frac{1}{2x10^{-5}\ f}$$

$$\Rightarrow \quad C_{eq} = 6.01x10^{-6}\ C$$

Problem 16.29

- Given the capacitor to the right:
  - What's E between the plates?
  - What's C?
  - What's the charge on each plate?



Problem 16.31

- Given the capacitor to the right:

a.) What's C?

$$C = \epsilon_0 \frac{A}{d} = (8.85x10^{-12})\frac{0.2\ m^2}{0.003\ m} = 5.9x10^{-10}\ \text{F or 590 pF}$$

b.) What's q on each plate?

$$Q = C\Delta V = (5.9x10^{-10}F)(6\ V) = 3.54x10^{-9}\ C$$

c.) What's E?

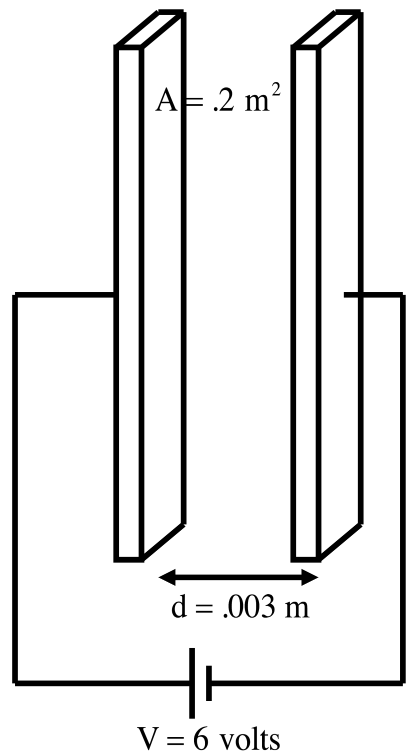
$$\vec{E} \cdot \vec{d} = -\Delta V$$
$$E(0.003\ m)(\cos 0) = -(0 - 6V)$$
$$E = 2000\ V/m$$

d.) What's q/A?

$$\frac{Q}{A} = \frac{3.54x10^{-9}\ C}{0.2\ m^2} = 1.77x10^{-8}\ C/m^2$$

e.) How will they all change if the plates are moved farther apart without disconnecting the voltage source?

If d increases, C decreases, so less q on each plate, weaker electric field between them, and less charge density.



Kirchhoff refresher

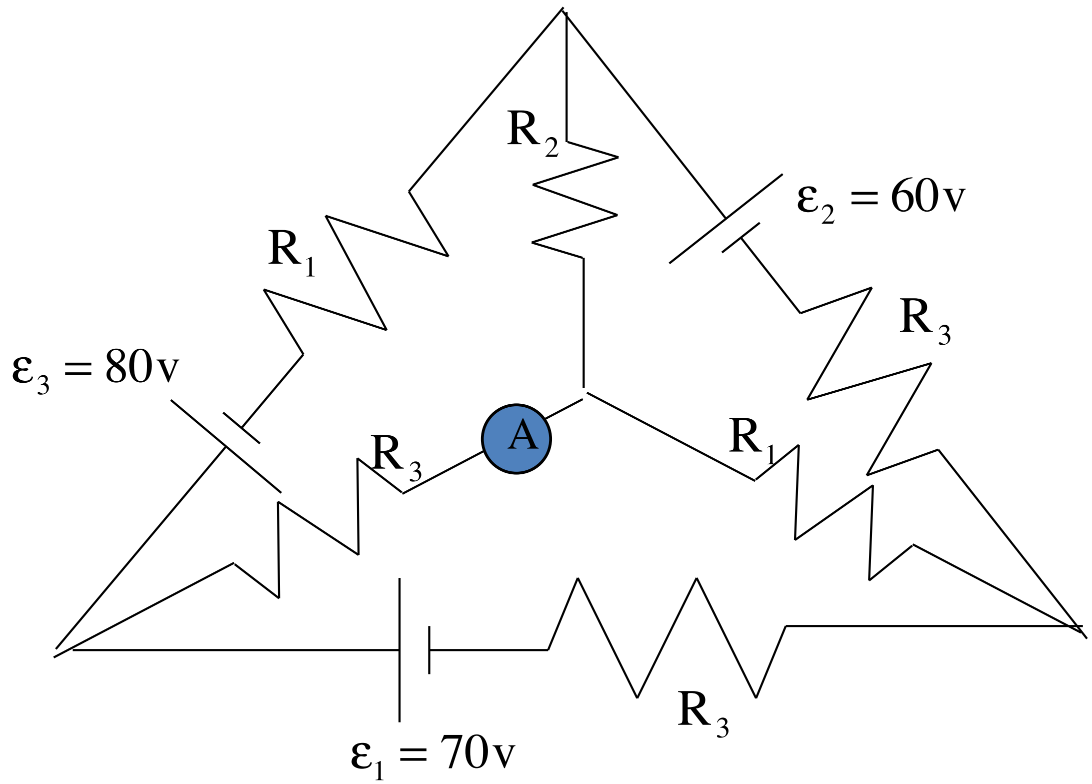
For the circuit to the right:

a) How many nodes? How many branches?

b) How many individual currents do you have to define in this circuit? How many equations do you need to solve?

c) Use Kirchhoff's Laws to write the equations you would need to solve for the current through the ammeter. YOU DO NOT NEED TO SOLVE THEM.

$R_1 = 2\ k\Omega \quad R_2 = 3\ k\Omega \quad R_3 = 4\ k\Omega$



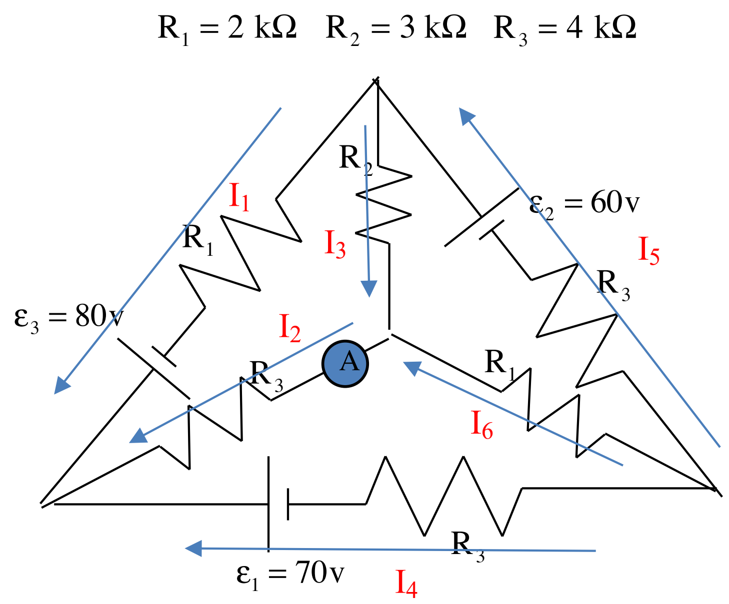
Kirchhoff refresher

For the circuit to the right:

a) How many nodes? How many branches?
   4 nodes, 6 branches

b) How many individual currents do you have to define in this circuit? How many equations do you need to solve?
   6 currents (or at least 3, with others in terms of those three). You need 6 equations for 6 unknowns.

c) Use Kirchhoff's Laws to write the equations you would need to solve for the current through the ammeter. (Many options, I'll do 3 nodes and 3 loops):



$I_5 = I_1 + I_3$	$80 + R_3I_2 + R_2I_3 - R_1I_1 = 0$
$I_6 + I_3 = I_2$	$60 - R_2I_3 + R_1I_6 - R_3I_5 = 0$
$I_1 + I_2 + I_4 = 0$	$70 + R_3I_2 + R_1I_6 - R_3I_4 = 0$